Derivation of the shadow system



(i)
$$Z(t,x) \rightarrow \xi(t) \text{ as } D \rightarrow \infty$$

Kondo and M.(2015)
(ii) $\xi_t = R\xi \left(\frac{1}{L} \int_0^L P dx - \frac{\mu}{L} \int_0^L X dx - 1\right)$

$$\begin{cases} P_t = P\left(1 - \frac{P + aX}{K} - Z\right) + P_{xx} \\ X_t = X\left(1 - \frac{X + bP}{K} - d(\mu)Z\right) + X_{xx} \\ Z_t = RZ(P - \mu X - 1) + DZ_{xx}, \\ P_x = X_x = Z_x = 0, \quad t > 0, x = 0, L \end{cases}$$

$$(SS) \begin{cases} P_{t} = P\left(1 - \frac{P + aX}{K} - \xi\right) + P_{xx} \\ X_{t} = X\left(1 - \frac{X + bP}{K} - d(\mu)\xi\right) + X_{xx}, & t > 0, \ 0 < x < L \\ \xi_{t} = R\xi\left(\frac{1}{L}\int_{0}^{L}Pdx - \frac{\mu}{L}\int_{0}^{L}Xdx - 1\right) \\ (P_{x}, X_{x})(t, 0) = (0, 0) = (P_{x}, X_{x})(t, L), & t > 0. \\ (P(0, x), X(0, x), \ \xi(0)) = (P_{0}(x), \ X_{0}(x), < Z_{0} >) \\ \text{spatial average of } Z_{0} \end{cases}$$