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Detecting Information Flows in Dominant Components of Stock Market Returns *

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Abstract

Political events and economic problems occurred in one country have increasingly influenced the economies and financial markets worldwide since the global economic crisis driven by the bankruptcy of Lehman Brothers in September 2008. For making both short-term and long-term investment strategies for investment firms and pension fund managements, and for planning financial and economic policies to control the crisis spillovers, it is indispensable to understand the fluctuation characteristics of financial markets that cause changes of the financial and economic environments in short-term and long-term periods. This article aims at detecting information flows for short-term and long-term investments by investigating the fluctuation relationships between dominant components of stock market return, based on long-term daily time series of TOPIX, S&P500 and DAX Index. We decompose each stock market return into two components, i.e., the return trend component and the return cyclical component, which are derived by applying a seasonal adjustment model proposed by Gersch and Kitagawa (1983) and Kitagawa and Gersch (1984, 1996) to the stock market index. The dominant component is found for each of short-term and long-term returns, respectively. Then, we investigate the relationships of fluctuation between dominant components of stock market return by conducting generalized power contribution analysis (Tanokura and Kitagawa 2004, 2015). The information flows of the influential factors are detected. Our findings indicate the importance of observing mutual relationships of long-term fluctuations, i.e., trends, between the three markets and can provide the useful information for building an investment strategy and making an economic policy.

Keywords: Generalized power contribution, Seasonal adjustment model, Return decomposition, Time series analysis, Long-term investment

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1 Introduction

Political events and economic problems occurred in one country have increasingly influenced the economies and financial markets worldwide since the global economic crisis driven by the bankruptcy of Lehman Brothers in September 2008. Under these circumstances, in Japan, having a serious problem of aging society in near future, the awareness of pension has largely increased. For making both short-term and long-term investment strategies for investment firms and pension fund managements, and for planning financial and economic policies to control the above-mentioned spillovers, it is indispensable to understand the fluctuation characteristics of financial markets that cause changes of the financial and economic environments in short-term and long-term periods. In particular, since the prices of financial markets fluctuate with both serial correlations and time series correlations, it is significant to evaluate the degree and direction of influence between markets.

This article aims at detecting information flows for short-term and long-term investments by investigating the fluctuation relationships between dominant components of stock market return. We focus on the analysis of three stock market indices such as TOPIX (Japan), DAX Index (Germany) and S&P 500 (US) for the period from January 4, 1993 to May 31, 2019 (6,890 days).

In financial market analysis, technical analysis is practically well known. Murphy defines technical analysis as the study of market action, primarily through the use of charts, for the purpose of forecasting future price trends (Murphy 1986). In a stock market, a trend of the stock price may be regarded as the gradually changing long-term fluctuations caused by characteristics specific to the stock. On the other hand, a cyclical fluctuations around the trend can sensitively be influenced by those of any other stock prices, and they may behave as only a price adjustment or may lead to a future change in the trend direction. Therefore, identifying a trend of price movement is significantly important.

We decompose each stock market return into two components, i.e., the return trend component and the return cyclical component. These components are derived by applying a seasonal adjustment model proposed by Gersch and Kitagawa (1983) and Kitagawa and Gersch (1984, 1996) to the stock market index. The dominant component is found for each of short-term and long-term returns. This implies the effectiveness of the seasonal adjustment model.

To evaluate the influence of external information on the stock markets for the last 26 years, we focus on two influential indices such as WTI crude oil futures which is often referred to as a proxy of the oil price, and JPYUSD which is the foreign exchange rate of the Japanese Yen against the US dollar. Then, we investigate the relationships of fluctuation between dominant components of stock market return by conducting generalized power contribution analysis (Tanokura and Kitagawa 2004, 2015). As a result, the influential factors are detected. Our findings indicate the importance of observing mutual relationships of long-term fluctuations, i.e., trends, between these markets. The information flows of the influential components of return detected by statistical modeling can provide the useful information for building an investment strategy and making an economic policy. As far as we know, there is no researches analyzing long-term financial time series by applying statistical methods.

This article consists of two parts. The following chapter introduces two statistical methods which we utilize in this article, such as a seasonal adjustment model and a generalized power contribution. Chapter 3 provides the empirical analysis for three stock market indices. Each stock market return is decomposed into the return trend component and the return cyclical component. And then, we detect the dominant component for each of short-term and long-term returns and investigate the influential factors in fluctuation of these returns. Finally, we state our conclusions and future works.

2 Statistical Modeling

2.1 A Seasonal Adjustment Model

This section briefly introduces a seasonal adjustment model proposed by Gersch and Kitagawa (1983) and Kitagawa and Gersch (1984, 1996).

For an observed time series y_n is expressed as

$$y_n = t_n + s_n + p_n + w_n. \tag{1}$$

A trend component t_n is estimated by the following trend component model with the trend order k

$$\Delta^k t_n = v_{n1}, \qquad v_{n1} \sim N(0, \tau_1^2). \tag{2}$$

A seasonal component s_n, which forms seasonal fluctuations if it exists, is expressed by the following seasonal component model with a period length p:

$$\sum_{i=0}^{p-1} s_{n-i} = v_{n2}, \qquad v_{n2} \sim N\big(0, \tau_2^2\big). \tag{3}$$

A stationary component p_n is estimated by the following stationary AR component model of AR order m:

$$p_n = \sum_{i=1}^{m} a_i p_{n-i} + v_{n3}, \qquad v_{n3} \sim N(0, \tau_3^2), \tag{4}$$

which expresses relatively shorter cyclical fluctuations than the gradual long-term trend component (2). Finally, the distribution of the observation noise w_n in (1) is given by

$$w_n \sim N(0, \sigma^2).$$

As each component model can be expressed in a state-space model form, the parameter estimations are performed through the composite form of state-space models. The details can be found in Kitagawa (2010).

2.2 Generalized Power Contribution

As a tool to detect ramifications of price fluctuations, we introduce a generalized power contribution which extends the concept of the Akaike's power contribution (Akaike 1968) by decomposing a variance covariance matrix of the noises.

Assume that an l-dimensional stationary time series $\mathbf{y}_n = (y_n(1), y_n(2), \cdots, y_n(l))^t$, $n = 1, \cdots, N$, is expressed as the following multivariate AR model with order m:

$$\mathbf{y}_n = \sum_{j=1}^{m} \mathbf{A}_j \mathbf{y}_{n-j} + \mathbf{v}_n, \tag{5}$$

where $\mathbf{A}_j$ is an $l \times l$ AR coefficient matrix with its (r, s)-component $a_j(r, s)$. An l-dimensional white noise $\mathbf{v}_n$ satisfies the following conditions:
$\mathrm{E}(\mathbf{v}_n) = [0, \cdots, 0]^t, \mathrm{E}(\mathbf{v}_n \mathbf{v}_n^t) = \mathbf{W}, \mathrm{E}(\mathbf{v}_n \mathbf{v}_h^t) = \mathbf{O}\ (n \neq h), \mathrm{E}(\mathbf{v}_n \mathbf{y}_h^t) = \mathbf{O}\ (n > h)$. Here, $\mathbf{O}$ is the $l \times l$ zero matrix, and $\mathbf{W} = (\sigma_{rs})$ is a symmetric positive definite matrix (i.e., $\sigma_{rs} = \sigma_{sr}$) that is referred to as the variance covariance matrix of the noises.

The cross spectrum matrix is defined as the $l \times l$ matrix $\mathbf{P}(f) = (P_{rs}(f))$, where the element $P_{rs}(f)$ is the Fourier transform of the cross-covariance function $C_k(r, s)$ and is referred to as the cross spectrum. Here, f is a frequency satisfying $-1/2 \leq f \leq 1/2$. The diagonal element $P_{rr}(f)$ is referred to as the power spectrum.

It is well known that $\mathbf{P}(f)$ can be obtained as

$$\mathbf{P}(f) = \mathbf{A}(f)^{-1} \mathbf{W} (\mathbf{A}(f)^{-1})^*, \tag{6}$$

where $\mathbf{A}(f)$ is the $l \times l$ complex matrix with its (r, s)-component $A_{rs}(f)$, and $\mathbf{A}^*$ denotes the complex conjugate of a matrix $\mathbf{A}$. Here, $A_{rs}(f)$ is defined as the Fourier transform of the coefficients $a_j(r, s)$ in the multivariate AR model (5):

$$A_{rs}(f) = \sum_{j=0}^{m} a_j(r, s) e^{-2\pi i j f}, \tag{7}$$

where $a_0(r, r) = -1$, and $a_0(r, s) = 0$ for $r \neq s$ (Whittle 1963; Akaike and Nakagawa 1988). For simplicity, denoting $\mathbf{A}(f)^{-1}$ as $\mathbf{B}(f) = (b_{rs}(f))$, (6) is given by

$$\mathbf{P}(f) = \mathbf{B}(f)\ \mathbf{W}\ \mathbf{B}(f)^*. \tag{8}$$

As the original definition in Akaike (1968) assumes that the components of the noise v_n are mutually uncorrelated: $\sigma_{rs} = 0,\ r \neq s$, $\mathbf{W}$ is diagonal. Therefore, from (8) the power spectrum of the r-th component $y_n(r)$ of $\mathbf{y}_n$ at a frequency f can be simply expressed as

$$P_{rr}(f) = \sum_{s=1}^{l} b_{rs}(f)\sigma_{ss}b_{rs}(f)^* \equiv \sum_{s=1}^{l} |b_{rs}(f)|^2\sigma_{ss}. \tag{9}$$

That is, the power spectrum $P_{rr}(f)$ of $y_n(r)$ is composed of l noise influences, and the degree of influence from the s-th noise component $v_n(s)$ on the fluctuation of $y_n(r)$ is evaluated by $|b_{rs}(f)|^2\sigma_{ss}$ for $s = 1,\ldots,l$. Therefore, Akaike's power contribution is defined as

$$r_{rs}(f) = \frac{|b_{rs}(f)|^2\sigma_{ss}}{P_{rr}(f)}, \tag{10}$$

which expresses the proportion of the fluctuation of $y_n(r)$ caused by the s-th noise component $v_n(s)$ at a frequency f.

For a general case, we consider decomposing $\mathbf{W}$ into a sum of matrices with rank one. We assume that the common influence of all the variables is derived from the smallest correlation coefficient. Then, $\mathbf{W}$ is expressed as a sum of at most $l(l+1)/2$ terms:

$$\mathbf{W} = \sum_{k=0}^{l-2}\sum_{j=1}^{k+1} q_{l-(k+1)+j,j}\ \mathbf{I}_{\mathbf{H}_j(k)}\mathbf{I}^t_{\mathbf{H}_j(k)} + \sum_{j=1}^{l} q_{jj}\ \mathbf{I}_{\mathbf{H}_j(l-1)}\mathbf{I}^t_{\mathbf{H}_j(l-1)}, \tag{11}$$

where $\mathbf{I}_{H_j(k)} = [i_{jk}(1),\ldots,i_{jk}(l)]$ is an l-dimensional vector, of which k components are 0 and $(l-k)$ components are either 1 or -1, depending on the signs of correlations for $k = 0,\ldots,l-1; j = 1,\ldots,k+1$. Here, $\mathbf{H}_j(k)$, the suffix of $\mathbf{I}_{\mathbf{H}_j(k)}$, is a subset $\mathbf{H}_j(k) = \{h_{j,1},\ \ldots,\ h_{j,k}\}$ of $\mathbf{H} = \{1,\ldots,l\}$ and indicates the components of 0 of $\mathbf{I}_{\mathbf{H}_j(k)}$. Note that the last term of (11) can be expressed as $\mathrm{diag}\{q_{11},\ldots,q_{ll}\}$.

Then, by (8),

$$\begin{aligned}\mathbf{P}(f) \quad = \quad & \sum_{k=0}^{l-2}\sum_{j=1}^{k+1} q_{l-(k+1)+j,j}\ \mathbf{B}(f)\ \mathbf{I}_{\mathbf{H}_j(k)}\mathbf{I}^t_{\mathbf{H}_j(k)}\ \mathbf{B}(f)^* \\ & + \mathbf{B}(f)\ \mathrm{diag}\{q_{11},\ldots,q_{ll}\}\ \mathbf{B}(f)^*.\end{aligned} \tag{12}$$

Therefore, the power spectrum of its r-th component is expressed as

$$\begin{aligned}P_{rr}(f) \quad = \quad & \sum_{k=0}^{l-2}\sum_{j=1}^{k+1} q_{l-(k+1)+j,j} \sum_{h=1,h\neq r}^{l}\ \sum_{n=1,n\neq r}^{l} c_{rjk}(h)c_{rjk}(n)^* \\ & + \sum_{j=1}^{l} q_{jj}|b_{rj}(f)|^2,\end{aligned} \tag{13}$$

where $c_{rjk}(h) = i_{jk}(h)\, b_{rh}(f)$.

(13) implies that the power spectrum $P_{rr}(f)$ can generally be decomposed into two terms. The first term expresses the $l(l-1)/2$ common influences resulting from correlations between l noise components, and the second one does the l influences resulting from the diagonal matrix of the noises. We refer to the first term as correlated noise and the second term as independent noise. Note that (13) becomes (9) when $q_{rs} = 0$ for $r \neq s$.

Finally, the generalized power contribution is defined as

$$\tilde{r}_{rjk}(f) = \begin{cases} \dfrac{q_{l-(k+1)+j,j} \displaystyle\sum_{h=1,h\neq r}^{l} \sum_{n=1,n\neq r}^{l} c_{rjk}(h)\, c_{rjk}(n)^*}{P_{rr}(f)} & (k = 0, \ldots, l-2; j = 1, \ldots, k+1) \\ \dfrac{q_{l-(k+1)+j,j}\, |b_{rj}(f)|^2}{P_{rr}(f)} & (k = l-1; j = 1, \ldots, l). \end{cases} \tag{14}$$

As the generalized power contribution simultaneously measures the degree of influence between various combinations of the noises, multi-directional causations between variables can be evaluated. The details can be found in Tanokura and Kitagawa (2015).

3 Empirical Analysis

3.1 Data

We use the time series data of three stock market indices such as TOPIX (Japan), DAX Index (Germany) and S&P 500 (US) for the period from January 4, 1993 to May 31, 2019 (6,890 days).

To evaluate the influence of external information on the stock markets for the last 26 years, we focus on two influential indices such as WTI crude oil futures which is often referred to as a proxy of the oil price, and JPYUSD which is the foreign exchange rate of the Japanese Yen against the US dollar. Data source is Bloomberg LP. Hereafter, we refer to TOPIX, DAX Index, S&P 500, WTI crude oil futures and JPYUSD as TPX, DAX, SP, WTI and JPY, respectively.

The missing price on a market holiday is taken the price on the previous trading day.

3.2 Return Decomposition

We apply the seasonal adjustment model proposed by Kitagawa and Gersch to each log-transformed variable. Here, we suppose no seasonality for each index. Therefore, the following decomposition is performed for each y_n:

$$\log y_n = t_n + p_n + w_n, \qquad w_n \sim N(0, \sigma^2). \tag{15}$$

We refer to the trend component t_n as KG-trend. Taking 1-day difference of (15),

$$\log y_n - \log y_{n-1} = t_n - t_{n-1} + p_n - p_{n-1} + w_n - w_{n-1}$$

is obtained. Therefore, the daily return r_n is decomposed as

$$r_n = rt_n + rc_n, \tag{16}$$

where $rt_n = t_n - t_{n-1}$ and $rc_n = (p_n - p_{n-1}) + (w_n - w_{n-1})$ is referred to as the return trend component and the return cyclical component, respectively.

For a long-term return, we define the k-day return $(k > 1)$ as

$$r_n(k) = \log y_n - \log y_{n-k}. \tag{17}$$

For example, we take $k = 260$ as 260-day return, i.e., 1-year return, $k = 780$ as 3-year return and $k = 1,300$ as 5-year return. Then, as $r_n(k) = r_n + r_{n-1} + \cdots + r_{n-k+1}$, it can be decomposed as

$$r_n(k) = \sum_{i=1}^{k-1}(rt_{n-i} + rc_{n-i}) = rt_n(k) + ct_n(k). \tag{18}$$

Here, we refer to $rt_n(k)$ and $rc_n(k)$ as k-day return trend component and k-day return cyclical component, respectively. Note that $rt_n(k)$ is the k-day difference of the KG-trend.

3.3 Characteristics of KG-trend

Figure 1 shows the decomposition of the log-transformed TPX. By minimizing AIC (Akaike 1998), the trend order two and the AR order two are selected. The top graph shows the log-transformed TPX and its KG-trend, the middle one shows the stationary AR component, and the bottom one does the noise. Although the KG-trend does not look like a smooth trend curve due to the data length of 26 years long, the smooth KG-trend can be found in the short-term period, as shown in Figure 2 for the detailed period of June 1, 2018 to December 24, 2018.

Similarly, the other four variables are decomposed respectively, as shown in Figure 3. The trend order two and the AR order two are also selected for all indices by AIC.

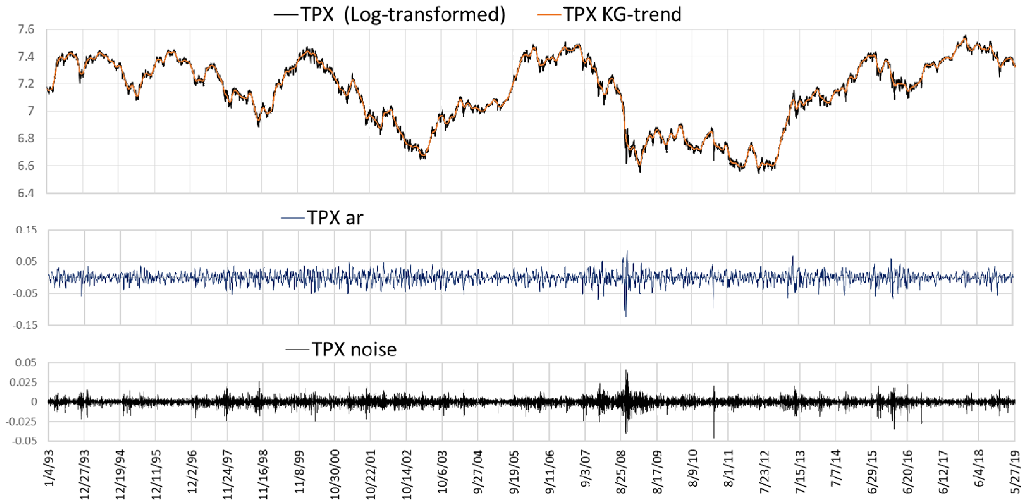

Figure 1: The decomposition of the log-transformed TOPIX (TPX): The index and the KG-trend (top), the stationary AR component (middle) and the noise (bottom).

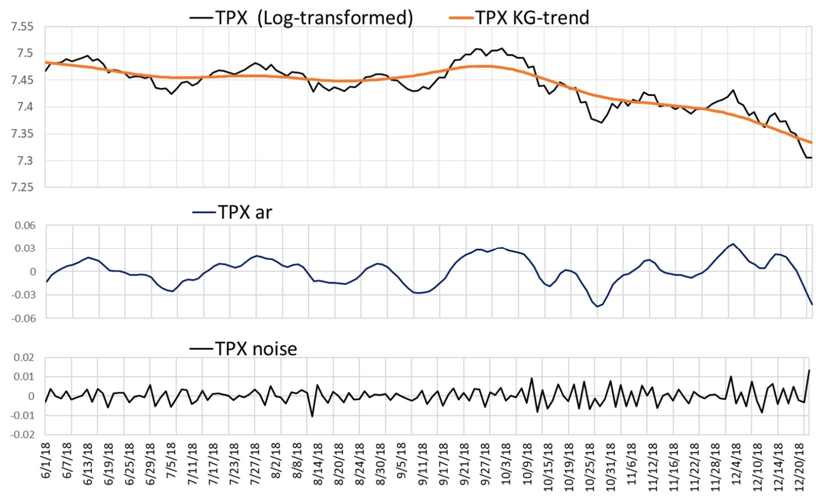

Figure 2: The log-transformed TOPIX(TPX) and the KG-trend for the detailed period of June 1, 2018 to December 24, 2018.

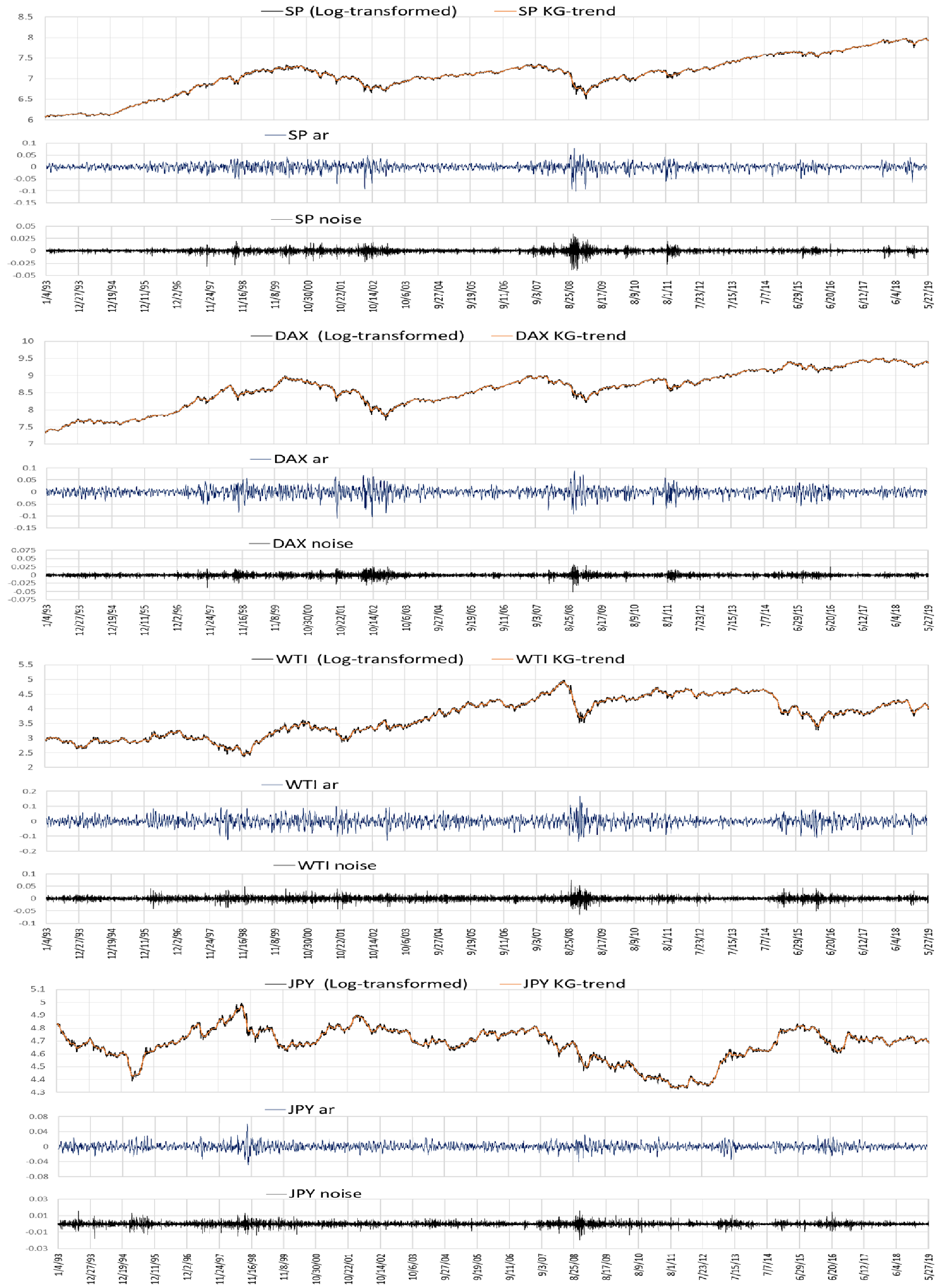

Figure 3: The decomposition of the log-transformed S & P 500(SP), DAX Index (DAX), WTI crude oil futures (WTI) and JPYUSD (JPY) from top to bottom.

Generally, stock prices always show zigzag movements even though they are in an up-trend or a down-trend. In order to indicate a relatively long-term trend of price movements, a simple moving average for the period of k-day is practically often used. However, as it is based on past prices, the longer k is for taking averages, the later a peak or a bottom of moving averages appears, which can be seen in the 20-day moving average (blue line) and the longer 60-day moving average (green line) for TPX in Figure 4. On the other hand, we found that the KG-trend (orange line) which is exponential-transformed corresponding to the original TPX, appropriately captures a peak or a bottom of the price movements. This is one of the merits of KG-trend.

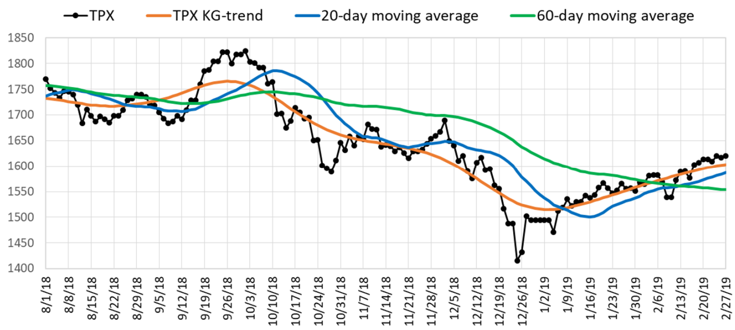

Figure 4: Comparison of a KG-trend with moving averages: the 20-day moving average (blue line), the 60-day moving average (green line) and the KG-trend (orange line) of TPX (black line).

As a KG-trend is estimated as a smoother which is fully utilized the price information during the whole period, it updates its own past values when a new information is added. Let us examine how a KG-trend changes. We compare the KG-trend for the whole period of January 4, 1993 to May 31, 2019 (6,890 days) with the one for Period C of January 4, 1993 to June 30, 2008 (4,041 days). In fact, the KG-trend for Period C was estimated based on the price information from January 4, 1993 to June 30, 2008.

The top graph in Figure 5 shows the KG-trend for the whole period (orange line) and the one for Period C (blue line) for TPX. As the differences between them in Period C are too small to distinguish the orange line from the blue one, the difference: the KG-trend for the whole period minus the KG-trend for Period C is shown in the bottom graph. The difference was in a significantly small range of $(-0.0005, 0.0005)$ until the end of May 1, 2008. As shown in the KG-trend for the whole period (orange line) of the top graph, the bankruptcy of Lehman Brothers occurred after Period C. As the KG-trend for Period C does not reflect this bankruptcy, since then, the difference has widen downward to -0.00559 at the end date of Period C.

Similarly, in Figure 6, the difference: the KG-trend for the whole period minus the KG-trend for Period C for SP, DAX, WTI and JPY is shown, respectively. It is intriguing that the difference for JPY has fallen in the small range of $(-0.003, 0.003)$ through Period C. It is because the KG-trend for the whole period exhibited the similar transitions with the KG-trend for Period C for a while after Period C. In fact, the sharp downtrend of JPY started at the middle August and lasted until the end of January, 2009.

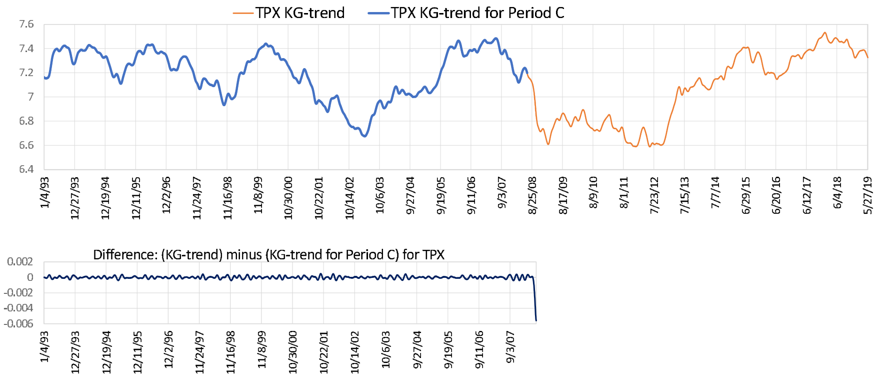

Figure 5: KG-trend for the whole period (orange line) and the one for Period C (January 4, 1993 to June 30, 2008; blue line) (top), and the difference: the KG-trend for the whole period minus the KG-trend for Period C (bottom) for TPX.

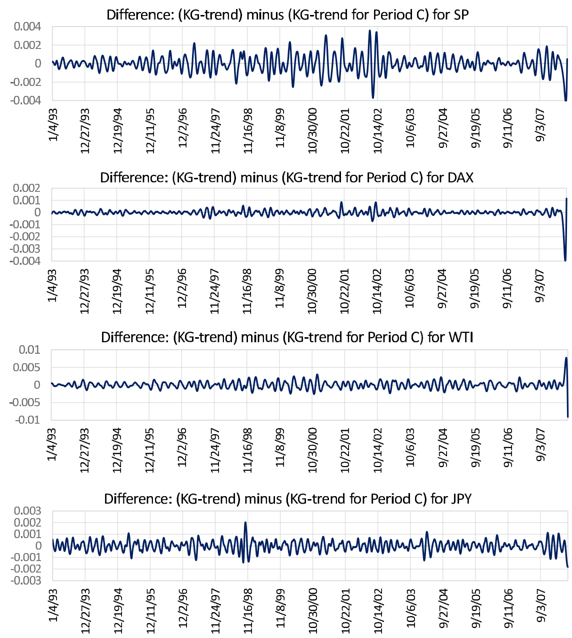

Figure 6: Difference: the KG-trend for the whole period minus the KG-trend for Period C for SP, DAX, WTI and JPY (from top to bottom).

From the point of view of long-term investment, the difference of KG-trends at the end date of the period is small enough to ignore. Therefore, in this sense, the stability of KG-trend is presumed.

3.4 Properties of Return Decomposition

Now we observe some outstanding properties on components of return as stated in (16) and (18). Figure 7 shows the return trend component (red line) and the return cyclical component (blue line) of the daily return for TPX, SP, DAX (left), WTI and JPY (right). It is found that the fluctuation range for return trend component is almost one tenth of that for return cyclical component for all indices. This implies the fluctuation of return cyclical component dominates that of daily return. In fact, the correlation coefficient between return cyclical component and daily return for TPX, SP, DAX, WTI and JPY is significantly high values of $0.985, 0.989, 0.985, 0.986$ and 0.983, respectively.

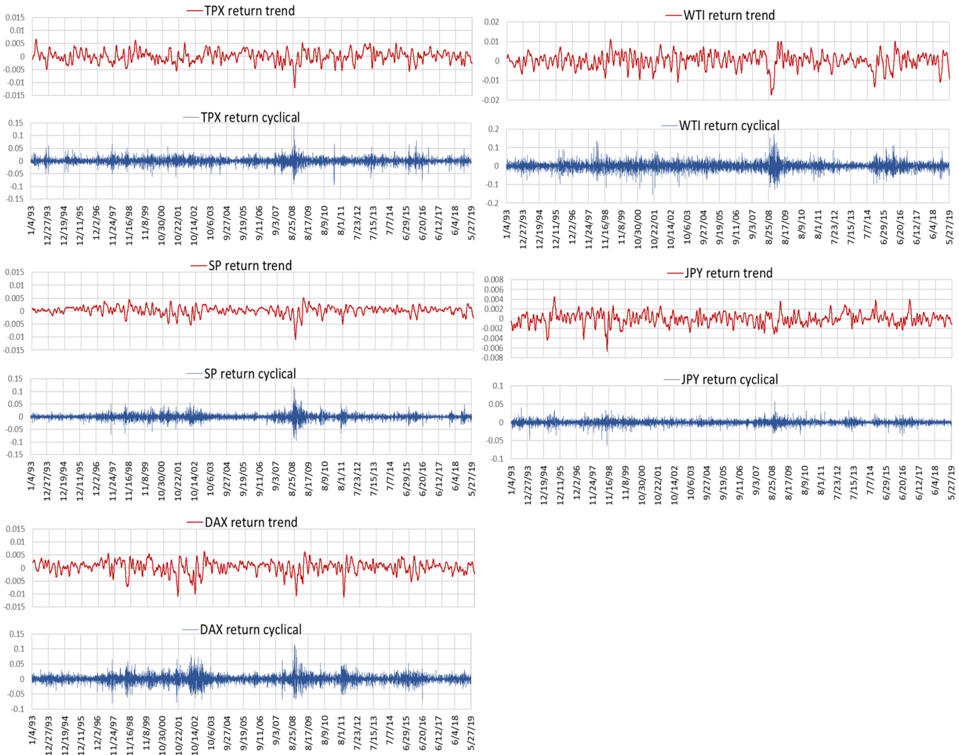

Figure 7: Return trend component (red line) and return cyclical component (blue line) of daily return for TPX, SP, DAX (left), WTI and JPY (right).

Let us consider the case of long-term returns. For example, Figure 8 shows the return trend component (red line) and the return cyclical component (blue line) of the 1-year (260-day) return for TPX, SP, DAX, WTI and JPY. Unlike the above-mentioned case of daily returns, the return trend component largely swings and the fluctuation range is larger than that of return cyclical component for all indices. In fact, the correlation coefficient between return trend component and 1-year return for TPX, SP, DAX, WTI and JPY significantly increases to $0.993, 0.992, 0.993, 0.991$ and 0.992, respectively. Also, those between return cyclical component and 1-year return for TPX, SP, DAX, WTI and JPY largely decreases to 0.152, 0.166, 0.154, 0.176 and 0.166, respectively.

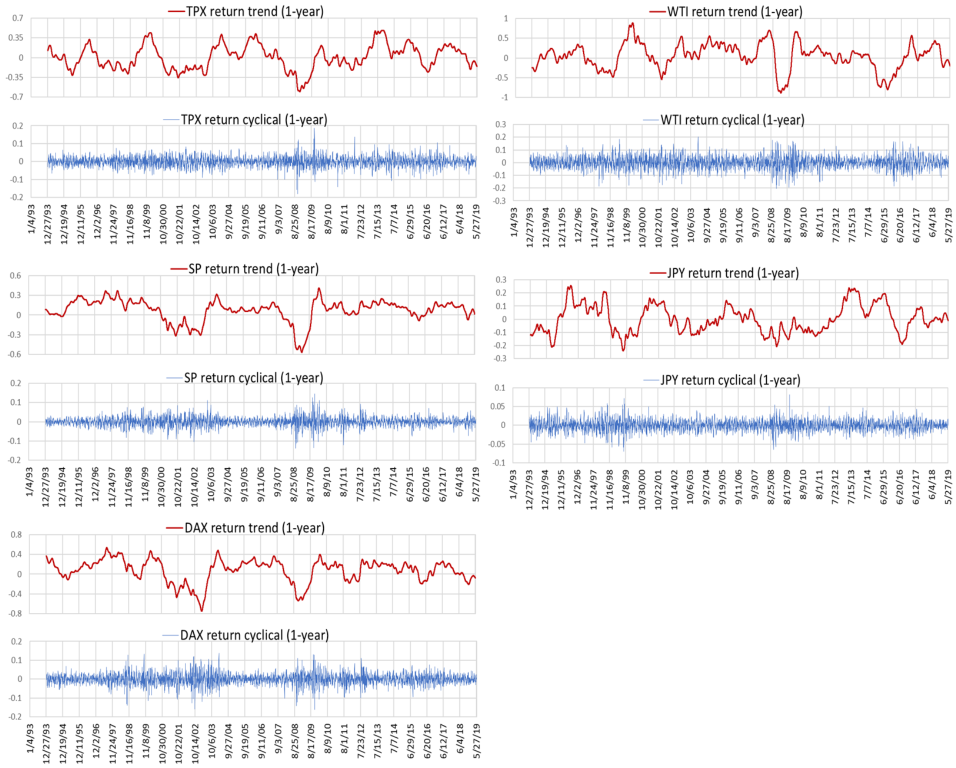

Figure 8: Return trend component (red line) and return cyclical component (blue line) of 1-year return for TPX, SP, DAX (left), WTI and JPY (right).

The next Table 1 compares the transition of correlation coefficients between return trend component and the return of 5-day, 20-day, 60-day, 3-year (780-day) and 5-year (1, 300-day) in the upper row, with that between return cyclical component and the each return for TPX, SP, DAX, WTI and JPY in the lower row. It is noteworthy that the longer period of return is, the larger the correlation coefficient of return trend component becomes. That is, long-term returns are closely related to the behavior of return trend component. Instead, the smaller the correlation coefficient of return cyclical component becomes. It is clarified that the return cyclical component plays an important role in fluctuations of daily return, while the trend component becomes more significant in fluctuations of longer term return.

Note that we do not consider the time difference between stock markets in this article. In practice, the New York market is open after the Tokyo stock market was closed on the same day. Table 2 compares the correlation coefficients of return trend component of 1-year return between markets. We divide the whole period of analysis into the four sub-periods as follows. Period C which stated before is divided into Period G: January 4, 1993 to October 17, 2000 and Period H: October 18, 2000 to June 30, 2008. The rest of the whole period is divided

Correlation coefficients with:	5-day return	20-day return	60-day return	3-year return	5-year return
TPX return trend component	0.515	0.874	0.966	0.998	0.998
SP return trend component	0.486	0.849	0.954	0.998	0.998
DAX return trend component	0.528	0.880	0.965	0.998	0.998
WTI return trend component	0.515	0.873	0.966	0.995	0.997
JPY return trend component	0.560	0.890	0.970	0.998	0.998
TPX return cyclical component	0.923	0.675	0.338	0.089	0.080
SP return cyclical component	0.937	0.725	0.393	0.092	0.077
DAX return cyclical component	0.920	0.674	0.347	0.091	0.082
WTI return cyclical component	0.924	0.677	0.339	0.128	0.104
JPY return cyclical component	0.911	0.663	0.320	0.092	0.086

Table 1: Correlation coefficient of return trend component with the return of 5-day, 20-day, 60-day, 3-year (780-day) and 5-year (1,300-day) for TPX, SP, DAX, WTI and JPY in the upper row, and that of return cyclical component with the return of 5-day, 20-day, 60-day, 3-year (780-day) and 5-year (1,300-day) for TPX, SP, DAX, WTI and JPY in the lower row.

into Period I: July 1, 2008 to February 15, 2013 and Period J: February 16, 2013 to May 31, 2019.

For reference, the lowest row shows the correlation coefficient between TPX and SP on the same day. Compared with the correlation coefficient between TPX and SP (1-day before), it is interesting that there were no significant differences between them. As a whole, the correlation coefficients reached the highest in Period I reflecting the spillover of crises. It is notable that the correlation coefficients of TPX with SP and DAX, were not so high in Period G, that is, the 1990's.

Correlation coefficients	Period G	Period H	Period I	Period J	Whole period
TPX and SP (1-day before)	0.299	0.535	0.813	0.712	0.590
DAX and TPX	0.281	0.563	0.757	0.701	0.558
DAX and SP	0.543	0.874	0.890	0.713	0.782
TPX and SP	0.295	0.532	0.815	0.701	0.587

Table 2: Correlation coefficient of return trend component of 1-year return between stock market indices: TPX and SP (1-day before), DAX and TPX, DAX and SP, and TPX and SP for Period G: January 4, 1993 to October 17, 2000, Period H: October 18, 2000 to June 30, 2008, Period I: July 1, 2008 to February 15, 2013 and Period J: February 16, 2013 to May 31, 2019.

On the other hand, the correlation coefficient of return cyclical component of daily return between stock market indices appears differently as shown in Table 3. The correlation coefficients between DAX and TPX were always the lowest level through all sub-periods. Unlike in the case of return trend component, compared the correlation coefficient between TPX and SP (1-day before) with that between TPX and SP (the lowest row), it is found that the time difference makes sense even though the correlation is relatively not so high. Similar with the case of return trend component in Table 2, the correlation coefficients reached the highest in Period I reflecting the spillover of crises.

Correlation coefficients	Period G	Period H	Period I	Period J	Whole period
TPX and SP (1-day before)	0.294	0.378	0.548	0.458	0.422
DAX and TPX	0.224	0.175	0.281	0.229	0.223
DAX and SP	0.290	0.572	0.672	0.504	0.526
TPX and SP	0.045	0.102	0.096	0.065	0.092

Table 3: Correlation coefficient of return cyclical component of daily return between stock market indices: TPX and SP (1-day before), DAX and TPX, DAX and SP, and TPX and SP for Period G: January 4, 1993 to October 17, 2000, Period H: October 18, 2000 to June 30, 2008, Period I: July 1, 2008 to February 15, 2013 and Period J: February 16, 2013 to May 31, 2019.

For daily investment, the time difference should be considered, however, we leave the further investigation for a future work.

3.5 Fluctuation Characteristics of Each Component

In the previous section, we clarify that the dominant component for daily returns is the return cyclical component and that for 1-year returns is the return trend component. In order to detect information flows of the above-mentioned dominant components, we investigate the fluctuation relationships of component between three stock market indices.

Now we conduct the generalized power contribution analysis to each component of daily return and 1-year (260-day) return, respectively. A five-variate AR model is fitted to each component of TPX, SP, DAX, WTI and JPY for each sub-periods of G: January 4, 1993 to October 17, 2000, H: October 18, 2000 to June 30, 2008, I: July 1, 2008 to February 15, 2013 and J: February 16, 2013 to May 31, 2019, and generalized power contributions are calculated.

Figure 9 shows the graph matrix of the generalized power contributions % of return trend component of 1-year return for three stock markets with power spectra. From top row to bottom row, the four sub-periods are shown, and from left column to right column, the generalized power contributions % of return trend component of 1-year return for TPX, SP and DAX. Each graph shows each proportion of contributor within the power spectrum (white line) at each frequency. For example, in Period G (the 1990s), the contribution from the index itself occupied $60-80$ % for TPX, while that for SP and DAX was only 20 %, respectively. Interestingly, this is the lowest contribution from the index itself among four subperiods for SP and DAX. This implies the global diversification already started in the US and Germany. Furthermore, from the middle graph in Period G, it is very impressive that TPX was involved in all correlated noise contributing to SP, and all the contributions are mostly constant at any frequency. On the other hand, from the top left graph, it is noteworthy that TPX was independent or self-contained in terms of fluctuations of 1-year return. The number of concerned indices of correlated noise such as TPX+DAX is mostly two for Period G, while more concerned indices of correlated noise contributed largely since then (i.e., the 2000s). There can be found some common noise contributions among countries and among sub-periods. As a whole, after the occurrence of the bankruptcy of Lehman Brothers in Period I, the contribution from various correlated noises increased.

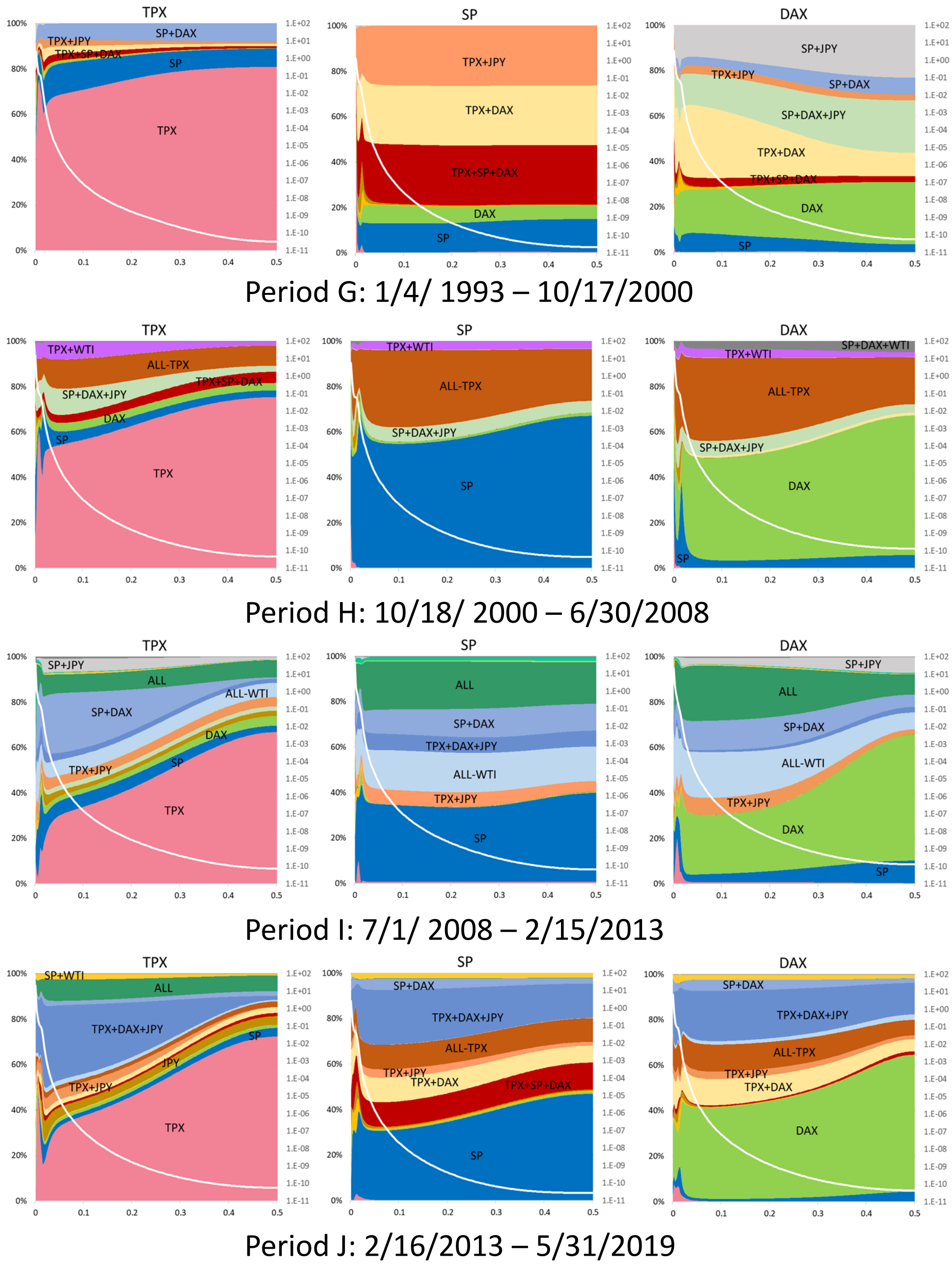

Figure 9: Generalized power contribution % and log-transformed power spectrum (rhs; white line) of return trend component of 1-year return for TPX, SP and DAX for the sub-periods of G, H, I and J (from top to bottom).

Next, consider the case of daily return. In Figure 10, the graph matrix of the generalized power contributions % of return cyclical component of daily return for three indices with power spectra are shown. From top row to bottom row, the four sub-periods are shown, and from left column to right column, the generalized power contributions % of return cyclical component of daily return for TPX, SP and DAX. Each graph shows each proportion of contributor within the power spectrum (white line) at each frequency. Compared with the previous case of 1-year return trend component, the magnitude of power spectrum significantly decreased for all indices according to the right hand scale. The contribution from index itself keeps consistently at least 20 % for all indices. In particular, in the current Period J, SP received the constantly 30 % self-contribution. This can be reflected by the recent bullish US stock market which has updated its highest record.

As a summary of the generalized power contribution analysis, Figure 11 shows the transitions of top 3 contributors with power contribution % of 1-year return trend component of TPX, SP and DAX (left) and those of daily return cyclical component of TPX, SP and DAX (right) for the sub-periods of G, H, I and J (from top to bottom). Each power contribution % is calculated for all frequency domain. For daily return cyclical component (right), the top 1 contributor was the index itself for all sub-periods. On the other hand, for 1-year return trend component (left), the top 1 were various kinds of noises. In Period G (the 1990s), for 1-year return trend component, it is astonishing that TPX was concerned with all three indices as not only the independent noise but also correlated noises. Then, in Period H (the 2000s), TPX lost its own contribution. This implies that the dependence of TPX on external information strengthened in fluctuations of 1-year return. Despite that, it is interesting that the independent noise of TPX largely contributed to other two indices. On the other hand, SP strongly contributed to all three indices, especially, DAX. As for daily return cyclical component (right), ALL-TPX largely contributed to all three indices in Period H. This implies that TPX was isolated in fluctuations of daily return. In Period I which was after the bankruptcy of Lehman Brothers, ALL-WTI largely contributed to TPX and DAX for 1-year return trend component (left), and to all three markets for return cyclical component (right). That is, WTI was isolated in fluctuations of both daily and 1-year returns. The contribution strength of SP remained in Period I. However, in the current Period J which was after the European debt crisis, the top 3 contributors were concentrated on DAX, TPX+DAX+JPY and TPX+DAX for all three markets for 1-year return trend component (left).

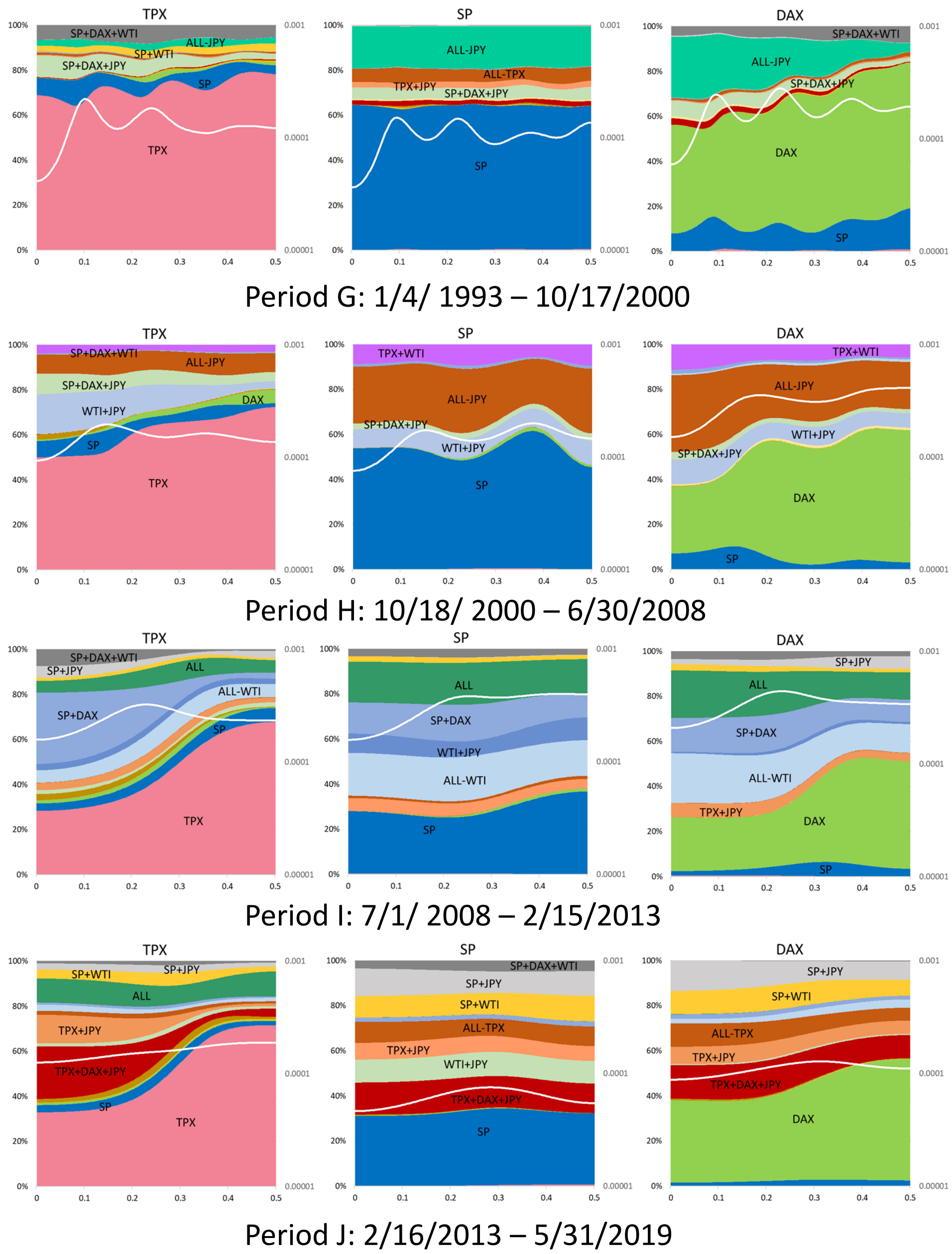

Figure 10: Generalized power contribution % and log-transformed power spectrum (rhs; white line) of return cyclical component of daily return for TPX, SP and DAX for the sub-periods of G, H, I and J (from top to bottom).

Return trend component (1-year)						Return cyclical component (1-day)					
Period G: 1/4/1993 - 10/17/2000						Period G: 1/4/1993 - 10/17/2000					
TPX		SP		DAX		TPX		SP		DAX	
TPX	51.07	TPX	36.48	TPX+DAX	26.98	TPX	72.12	SP	63.73	DAX	56.34
WTI	11.67	TPX+SP+DAX	12.34	TPX+SP+DAX	14.32	SP+DAX+WTI	6.55	ALL-JPY	18.44	ALL-JPY	16.77
DAX	6.79	TPX+JPY	12.34	TPX+JPY	14.32	SP	6.53	ALL-TPX	6.00	SP	12.08
Period H: 10/18/2000 - 6/30/2008						Period H: 10/18/2000 - 6/30/2008					
TPX		SP		DAX		TPX		SP		DAX	
SP	27.68	SP	37.05	SP	25.43	TPX	61.33	SP	53.47	DAX	49.47
WTI	23.32	DAX	31.01	TPX	21.61	WTI+JPY	10.74	ALL-TPX	24.93	ALL-TPX	22.68
ALL-WTI	14.53	TPX	17.62	DAX	18.99	ALL-TPX	6.58	WTI+JPY	8.78	WTI+JPY	7.93
Period I: 7/1/2008 - 2/15/2013						Period I: 7/1/2008 - 2/15/2013					
TPX		SP		DAX		TPX		SP		DAX	
ALL-JPY	21.66	SP	20.46	ALL-WTI	23.77	TPX	46.09	SP	29.53	DAX	33.68
SP+DAX+WTI	15.64	JPY	19.98	SP	22.59	SP+DAX	15.04	ALL-WTI	18.12	ALL-WTI	17.10
ALL-WTI	15.48	WTI	15.19	TPX+DAX+JPY	16.75	ALL-WTI	7.34	ALL	17.29	ALL	16.32
Period J: 2/16/2013 - 5/31/2019						Period J: 2/16/2013 - 5/31/2019					
TPX		SP		DAX		TPX		SP		DAX	
TPX+DAX+JPY	25.36	TPX+DAX	16.68	DAX	24.96	TPX	51.89	SP	32.46	DAX	43.38
DAX	18.26	DAX	12.64	TPX+DAX+JPY	17.77	TPX+DAX+JPY	11.12	TPX+DAX+JPY	13.98	TPX+DAX+JPY	13.46
TPX+DAX	13.77	SP	12.01	TPX+DAX	15.57	ALL	9.82	SP+JPY	10.54	SP+JPY	10.20

Figure 11: Top 3 contributors with power contribution % of return trend component of 1-year return of TPX, SP and DAX (left) and those of return cyclical component of daily return of TPX, SP and DAX (right) for the sub-periods of G, H, I and J (from top to bottom).

In Period J, the outstanding contribution of TPX for 1-year return trend component may be recovered as the contribution of SP to DAX disappeared. Moreover, the correlated noise of TPX+DAX+JPY commonly contributed to both 1-year return trend component and daily return cyclical component.

Our findings indicate the importance of observing the mutual relationships of long-term fluctuations, i.e., trends, between these markets. The information flows of influential component to return detected by statistical modeling can be useful in building an investment strategy and making an economic policy.

4 Conclusions

Aiming at detecting information flows for short-term and long-term investments by investigating the fluctuation relationships between dominant components of stock market return, we conducted the analysis on the three stock market indices such as TOPIX (Japan), DAX Index (Germany) and S&P 500 (US) for the period from January 4, 1993 to May 31, 2019 (6,890 days).

Compared with moving averages in practical technical analysis, we showed one of the merits of the KG-trend which is extracted from a stock market index by a seasonal adjustment model proposed by Gersch and Kitagawa (1983) and Kitagawa and Gersch (1984, 1996). In fact, we decomposed each stock market return into the return trend component and the return cyclical component which were derived by applying the seasonal adjustment model to the stock market index. It was found that the return cyclical component was dominant in daily return and so was the return trend component in 1-year return. This implies the effectiveness of applying the seasonal adjustment model. Then, to evaluate the influence of external information on the stock markets for the last 26 years, we focused on two influential indices such as WTI crude oil futures which is often referred to as a proxy of the oil price, and JPYUSD which is the foreign exchange rate of the Japanese Yen against the US dollar. We investigated the relationships of fluctuation between dominant components by conducting generalized power contribution analysis (Tanokura and Kitagawa 2004, 2015).

In the 1990s, for return trend component of 1-year return, TPX was concerned with all three market indices not only as the independent noise but also as correlated noises. Then, in the 2000s, TPX lost its own contribution, however, the independent noise of TPX largely contributed to other two markets. This implies that TPX strengthened the dependency on external information but remained isolated in terms of 1-year return fluctuation. On the other hand, in the 2000s, SP strongly contributed to all three indices in fluctuations of 1-year return.

As for return cyclical component of daily return, ALL-TPX largely contributed to all three indices in the period of early 2000s before the Lehman shock. This implies that TPX was isolated even in fluctuations of daily return. In the period before the bankruptcy of Lehman Brothers, ALL-WTI largely contributed to TPX and DAX for return trend component of 1-year return, and to all three indices for return cyclical component of daily return. This reflects the current oil market situation. Because WTI were isolated in fluctuations of both daily and 1-year returns. The contribution strength of SP remained after the Lehman shock. However, in the current period after the European debt crisis, the top 3 contributors were concentrated on DAX, TPX+DAX+JPY and TPX+DAX for all three markets for return trend component of 1-year return. As the contribution of SP to DAX disappeared and the correlated noise of TPX+DAX+JPY commonly contributed to both return trend component and return cyclical component, the strength of SP contribution decreased. The outstanding contribution of TPX for return trend component of 1-year return in the 1990s may be recovered.

Our findings in this article indicate the importance of observing the mutual relationships of long-term fluctuations which is trends, between these stock markets. The information flows of influential component to return fluctuations detected by statistical modeling can provide the useful information for building an investment strategy and making an economic policy.

Finally, following future works can be considered. As a KG-trend is estimated as a smoother which is fully utilized the price information during the whole period, it updates its own past values when a new information is added. Therefore, the existence of the difference between the past KG-trend and the updated KG-trend close to the end of the past period will be investigated. In addition, the influence of the time difference between the New York market and the Tokyo stock market on the same day should be investigated. Utilizing the information flows, the asset allocation between three markets can be further analyzed. This can be practically useful.

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